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BRIEF COMMUNICATION

CONTINUOUS MEASUREMENT OF THE DENSITY OF FLOWING SLURRIES

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Figure 1 shows schematically a device which is commonly used to determine the density and hence concentration of a flowing slurry. It consists essentially of a U-loop with the pressure difference determined between two stations, 1 and 2, in the riser limb A, and similarly between stations 3 and 4 in the downcomer B. Such a device was first proposed by Hagler (1956). Assessments of the technique have been given by Brook (1962), Einstein & Graf (1966) and Weisman & Graf (1968), all of whom also considered its use for measuring slurry flow rate. Similarly, Wilson (1973) used such a loop to infer the frictional pressure loss associated with flow of a fully-suspended slurry. The general consensus is that, while a U-loop gives reliable measurements of slurry density, values for flow-rate or frictional loss are less reliable. The present note sets out an analysis of the behaviour of such a device which is more complete than those given previously, and applies the conclusions to delineate conditions under which errors may be anticipated.

ANALYSIS

The only analytical complication arises from the fact that, because of settling of solids relative to the liquid in the slurry, the *in situ* solid concentration in the riser is greater than that in the downcomer. Hagler (1956), Brook (1962) and Herringe (1977) assumed that the delivered concentration is the mean of these two values, while Einstein & Graf (1966) and Weisman & Graf (1968) effectively assumed that the settling velocity of the solids is the same in each limb. Neither of these assumptions is strictly valid: the higher solids concentration in the riser leads to a lower settling velocity in that limb. The effect can be examined in terms of the relationship

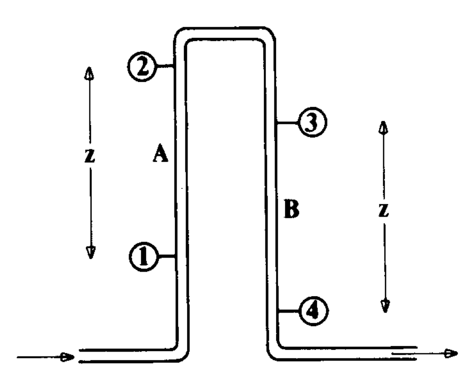

Figure 1. Measurement loop: schematic.

between hindered settling velocity and concentration developed by Richardson & Zaki (1954). Leung *et al.* (1969) showed that the Richardson–Zaki correlation is applicable to vertical hydraulic conveying provided that the solids concentration by volume does not exceed 50–55 per cent. The solids slip with their hindered settling velocity, v_s, relative to the mean mixture velocity (i.e. total volumetric discharge rate divided by pipe cross-sectional area), V_m. Thus in the riser limb, A, where the volumetric solids concentration is C_{VA}, the average solids velocity is

$$V_{SA} = V_m - v_{SA} \quad [1]$$

where v_{SA} is the hindered settling velocity at concentration C_{VA}. Solids continuity then yields

$$C_{Vd} V_m = C_{VA} V_{SA} = C_{VA}(V_m - v_{SA}) \quad [2]$$

where C_{Vd} is the solids concentration in the delivered slurry. Similarly, for the downcomer, B,

$$C_{Vd} V_m = C_{VB}(V_m + v_{SB}) . \quad [3]$$

Richardson & Zaki (1954) showed that C_V and v_s are related by

$$v_{SA} = v_t(1 - C_{VA})^n; \quad v_{SB} = v_t(1 - C_{VB})^n \quad [4]$$

where n is an empirical index dependent on the Reynolds number of a single particle falling at the terminal velocity, v_t (see below). Equations [2]–[4] can be solved to yield C_{VA} and C_{VB}. The mean density of the slurry in the two limbs is given by

$$\rho_{mA} = \rho_L + C_{VA}(\rho_s - \rho_L); \quad \rho_{mB} = \rho_L + C_{VB}(\rho_s - \rho_L) \quad [5]$$

where ρ_L and ρ_s are the densities of liquid and solid.

We now consider the pressure difference between the two measuring stations 1 and 2, distant z apart, in the riser. Provided that flow is fully developed,

$$P_1 - P_2 = zg\rho_{mA} + 4\tau_A z/D \quad [6]$$

where τ_A is the wall shear stress and D the pipe diameter. Similarly, for limb B,

$$P_4 - P_3 = zg\rho_{mB} - 4\tau_B z/D . \quad [7]$$

The values of τ correspond to turbulent flow of a pseudo-homogeneous suspension, and are very weakly dependent on solids concentration (Carstens & Addie 1981). Since the mixture velocity is the same in each limb, we may make the reasonable assumption that $\tau_A = \tau_B = \tau$. Adding [6] and [7] then yields

$$\rho_{mA} + \rho_{mB} = [(P_1 - P_2) + (P_4 - P_3)]/zg . \quad [8]$$

The equivalent result in terms of solids concentration follows from [5] and [8]:

$$C_{VA} + C_{VB} = \left[\frac{(P_1 - P_2) + (P_4 - P_3)}{zg} - 2\rho_L\right] \times \left(\frac{1}{\rho_s - \rho_L}\right). \quad [9]$$

Thus the average pressure gradient does indicate the average of the *in situ* densities and concentrations. It remains to determine how closely these averages represent the properties of the delivered slurry. Equations [2] and [3] may be rearranged using binomial expansions (see Appendix) to give

$$C_{VA}+C_{VB}=C_{vd}\left[2+\frac{(v_{SA}-v_{SB})}{V_m}+\frac{(v_{SB}^2+v_{SB}^2}{V_m^2}+\dots\right] \qquad [10]$$

which shows that the error in measurement can be dominated by the difference in settling velocity between the two limbs. To put 10 into a more convenient form, v_{SA} and v_{SB} are related to the hindered settling velocity, v_{Sd}, at delivered concentration by applying a Taylor expansion. Manipulations are given in the Appendix. It turns out that both correction terms in [10] are of order $(v_{Sd}/V_m)^2$ and, to this order,

$$C_{VA}+C_{VB}=2C_{Vd}\left[1+\frac{v_{Sd}^2}{V_m^2}\left\{1-\frac{nC_{Vd}}{1-C_{Vd}}\right\}\right] \qquad [11]$$

where n is the Richardson–Zaki index.

Use of a U-loop to determine flow rate relies on determination of τ. Subtracting [7] from [6], with the assumption $\tau_A=\tau_B=\tau$, gives

$$\tau=\frac{D}{8}\left[\frac{(P_1-P_2)-(P_4-P_3)}{z}-g(\rho_{mA}-\rho_{mB})\right] \qquad [12]$$

or, from [5],

$$\tau=\frac{D}{8}\left[\frac{(P_1-P_2)-(P_4-P_3)}{z}-g(\rho_s-\rho_L)(C_{VA}-C_{VB})\right]. \qquad [13]$$

The density term in [12] and [13] represents the error involved in estimating τ from the difference between the pressure drop measurements. To order consistent with [11], [13] becomes (see Appendix).

$$\tau=\frac{D}{8}\left[\frac{(P_1-P_2)-(P_4-P_3)}{z}-2g(\rho_s-\rho_L)C_{Vd}\frac{v_{Sd}}{V_m}\right]. \qquad [14]$$

Thus the error in evaluation of τ is of order (v_{Sd}/V_m), compared to $(v_{Sd}/V_m)^2$ for evaluation of density or concentration, and is equivalent to the result developed by Einstein & Graf (1966).

It may also be noted that, whereas the error in density or concentration may be in either direction, the difference between pressure drops always gives a value of τ which is too high. To estimate the relative magnitude of this term, an estimate for τ is necessary. A good estimate may be obtained from the value for liquid alone flowing at mean velocity V_m (Carstens & Addie 1980):

$$\tau=f\rho_L V_m^2/2 \qquad [15]$$

where f is the fanning friction factor. The fractional error is then

$$\epsilon_\tau=\frac{Dg(\rho_s-\rho_L)C_{Vd}v_{Sd}}{4\tau V_m}=\frac{D}{fV_m^3}\left[\frac{gC_{vd}v_{Sd}(\rho_s-\rho_L)}{2\rho_L}\right]. \qquad [16]$$

DISCUSSION

The above analysis confirms the conclusion of earlier workers that a simple U-loop can give accurate measurements of slurry density and concentration, but estimates of shear stress or, by inference, velocity are much less reliable. Equations [11] and [16] enable estimation of the error

Table 1. Values for particles in water at 298 K: $\rho_L = 997$ kg.m^{-3}, $\mu = 8.94 \times 10^{-4}$ Nsm^{-2}

	Particle diameter, (m)	Terminal velocity, v_t(m/s)	Settling index, n	Hindered settling velocity, v_{Sd}(ms^{-1})			$v_{Sd}^2 \left[\frac{1-nC_{Vd}}{1-C_{Vd}}\right]$ (m^{2}s^{-2})			$\frac{gC_{Vd}v_{Sd}(\rho_s-\rho_\ell)}{2\rho_\ell}$ (m^{2}s^{-3})		
				C_{Vd} = .1	.2	.3	C_{Vd} = .1	.2	.3	C_{Vd} = .1	.2	.3
a)	Particle density = 2650 kg.m^{-3}											
	10^{-4}	9×10^{-3}	4.4	5.7×10^{-3}	3.4×10^{-3}	1.9×10^{-3}	1.6×10^{-5}	-1.1×10^{-6}	-3.1×10^{-6}	4.6×10^{-3}	5.5×10^{-3}	4.6×10^{-3}
	2×10^{-4}	2.7×10^{-2}	3.7	1.8×10^{-2}	1.2×10^{-2}	7.2×10^{-3}	1.9×10^{-4}	1.1×10^{-5}	-3.0×10^{-5}	1.5×10^{-2}	2.0×10^{-2}	1.8×10^{-2}
	5×10^{-4}	8.4×10^{-2}	3.0	6.1×10^{-2}	4.3×10^{-2}	2.9×10^{-2}	2.5×10^{-3}	4.6×10^{-4}	-2.3×10^{-4}	5.0×10^{-2}	7.0×10^{-2}	7.0×10^{-2}
	10^{-3}	0.17	2.6	0.13	9.2×10^{-2}	6.5×10^{-2}	1.1×10^{-2}	3.0×10^{-3}	-5.1×10^{-4}	0.11	0.15	0.16
	2×10^{-3}	0.28	2.4	0.22	0.16	0.12	3.5×10^{-2}	1.1×10^{-2}	-4.1×10^{-4}	0.18	0.26	0.29
	5×10^{-3}	0.5	2.4	0.39	0.29	0.21	0.11	3.4×10^{-2}	-1.3×10^{-3}	0.32	0.47	0.52
	10^{-2}	0.73	2.4	0.56	0.43	0.31	0.23	7.2×10^{-2}	-2.7×10^{-3}	0.46	0.70	0.75
b)	Particle density = 5000 kg.m^{-3}											
	10^{-4}	1.9×10^{-2}	4.1	1.3×10^{-2}	7.8×10^{-3}	4.5×10^{-3}	8.7×10^{-5}	-1.1×10^{-6}	-1.5×10^{-5}	2.5×10^{-2}	3.1×10^{-2}	2.7×-10^{-2}
	2×10^{-4}	5.2×10^{-2}	3.4	3.6×10^{-2}	2.4×10^{-2}	1.5×10^{-2}	7.9×10^{-4}	7.9×10^{-5}	-1.1×10^{-4}	7.1×10^{-2}	9.4×10^{-2}	8.9×10^{-2}
	5×10^{-4}	0.15	2.8	0.11	8.2×10^{-2}	5.6×10^{-2}	8.9×10^{-3}	2.0×10^{-3}	-6.6×10^{-4}	0.22	0.32	0.33
	10^{-3}	0.28	2.5	0.22	0.16	0.12	3.4×10^{-2}	1.0×10^{-2}	-8.3×10^{-4}	0.43	0.64	0.69
	2×10^{-3}	0.46	2.4	0.36	0.27	0.19	9.3×10^{-2}	2.9×10^{-2}	-1.1×10^{-3}	0.70	1.1	1.2
	5×10^{-3}	0.81	2.4	0.63	0.47	0.34	0.29	8.9×10^{-2}	-3.3×10^{-3}	1.2	1.9	2.0
	10^{-2}	1.1	2.4	0.85	0.64	0.46	0.53	0.16	-6.1×10^{-3}	1.7	2.5	2.7

to be expected. Table 1 summarises calculations for particles of density 2650 and 5000 kg.m^{-3} conveyed by water at 298 K. Terminal velocities are calculated from standard correlations (Clift *et al.* 1978), assuming the particles to be spherical. The index n is evaluated for closely-sized particles; for solids containing a range of sizes, these values should be increased slightly (Mirza & Richardson 1979).

From [11], the fractional error incurred in estimating delivered slurry density or concentration from the pressure gradient in the two limbs is $[v_{Sd}^2\{1 - nC_{vd}/(1 - C_{vd})\}/V_m^2]$. Table 1 gives values of $v_{Sd}^2\{1 - nC_{vd}/(1 - C_{vd})\}$, and shows that the error increases with particle size and density but decreases with increasing solids concentration, i.e. the effect of concentration is in the opposite sense to that predicted by the simplified analyses given by earlier authors. Typical conveying velocities for thses materials are of order 3 ms^{-1} or greater (Carstens & Addie 1980), so that V_m^2 is of order 10 m^2 s^{-2} and the magnitude of the fractional error is indicated by dividing the values of $v_{sd}^2\{1 - nC_{vd}/(1 - C_{vd})\}$ in Table 1 by 10. At a solids volume concentration of 30 per cent , the error is always negligible; at 20 per cent solids, the error is around 1 per cent for particles 100 mm in diameter; at 10 per cent solids, the error is around 1 per cent for 5-mm particles with density 2650 kg.m^{-3} and for 2-mm particles with density 5000 kg.m^{-3}.

The fractional error incurred in estimating the wall shear stress, τ, is given by [16]. Table 1 gives values of $[gC_{vd}v_{Sd}(\rho_s - \rho_L)/\rho_L]$, i.e. $\epsilon_\tau \times fV_m^3/D$. The error increases with particle size and density, and increases weakly with solids concentration. Typical values are $f \simeq 0.005$ and $D \simeq 0.2$ m, so that fV_m^3/D is of order unity and $[gC_{Vd}v_{Sd}(\rho_s - \rho_L)\rho_L]$ indicates the order of the fractional error. For solids with the density of sand (2650 kg^{-3}) the error reaches 1 per cent for particles little larger than 100 μm, and exceeds 10 per cent for 1-mm particles.

Thus we conclude that a U-loop of the type shown in figure 1 can be used to give reliable measurements of slurry density and concentration under most conditions of practical interest, but generally cannot be used to measure wall shear stress or mixture velocity. The analysis assumes that flow in the measurement sections is fully developed, which requires that they be located many pipe diameters downstream from the bends in the loop. Since the theoretical errors are so small, this practical consideration is of more concern is determining slurry density by differential pressure measurements.

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APPENDIX

Simplification of equations

The hindered settling velocity at concentration C_{VA} is related to that at the delivered concentration C_{Vd} by a Taylor expansion about C_{Vd}:

$$v_{SA} = v_{Sd} + (C_{VA} - C_{Vd})v'_S + \frac{1}{2}(C_{VA} - C_{Vd})^2 v''_s + \ldots \qquad [A1]$$

where primes denote derivatives with respect to C_V evaluated at C_{Vd}. Subtracting the corresponding result for v_{SB} from [A1] yields

$$v_{SA} - v_{SB} = (C_{VA} - C_{VB})v'_S + \frac{1}{2}(C_{VA} - C_{VB})(C_{VA} + C_{VB} - 2C_{Vd})v''_S + \ldots \qquad [A2]$$

Also from [A1],

$$v^2_{SA} = v^2_{Sd} + 2v_{Sd}[(C_{VA} - C_{Vd})v'_S + \ldots] \qquad [A3]$$

which may be combined with the corresponding result for v^2_{SB} to give

$$v^2_{SA} + v^2_{SB} = 2v_{Sd}[v_{Sd} + (C_{VA} + C_{VB} - 2C_{Vd})v'_S + \ldots]. \qquad [A4]$$

Applying binomial expansions to [2] and [3]

$$C_{VA} = C_{Vd}\left[1 + \frac{v_{SA}}{V_m} + \frac{v^2_{SA}}{V_m^2} + \ldots\right] \qquad [A5]$$

$$C_{VB} = C_{Vd}\left[1 - \frac{v_{SB}}{V_m} + \frac{v^2_{SB}}{V_m^2} + \ldots\right] \qquad [A6]$$

so that

$$C_{VA} - C_{VB} = C_{Vd}\left[\frac{v_{SA} + v_{SB}}{V_m} + \frac{v^2_{SA} - v^2_{SB}}{V_m^2} + \ldots\right] \qquad [A7]$$

$$C_{VA} + C_{VB} - 2C_{Vd} = C_{Vd}\left[\frac{v_{SA} - v_{SB}}{V_m} + \frac{v^2_{SA} + v^2_{SB}}{V_m^2} + \ldots\right]. \qquad [A8]$$

Equation [A8] is equivalent to [10]. Combining [A2], [A7] and [A8] and eliminating terms of higher than second order in (v_{Sd}/V_m) yields

$$\frac{v_{SA} - v_{SB}}{V_m} = \frac{2C_{Vd}v_{Sd}v'_S}{V_m^2}. \qquad [A9]$$

Similarly, from [A4] and [A8]

$$\frac{v^2_{SA} + v^2_{SB}}{V_m^2} = \frac{2v^2_{Sd}}{V_m^2}. \qquad [A10]$$

From the Richardson–Zaki relationship [4],

$$v_s' = -nv_{Sd}/(1 - C_{Vd}) \qquad \text{[A11]}$$

so that [A9] becomes

$$\frac{v_{SA} - v_{SB}}{V_m} = \frac{-2v_{SD}^2\, nC_{Vd}}{V_m^2(1 - C_{Vd})}. \qquad \text{[A12]}$$

Substitution of [A10] and [A12] into [10] yields [11].

For the error in estimation of τ, we require $(C_{VA} - C_{VB})$. From [A7], [A1] and [A3],

$$C_{VA} - C_{VB} = 2C_{Vd}\left[\frac{v_{Sd}}{V_m} + 0\left\{\left(\frac{v_S}{V_m}\right)^3\right\}\right]. \qquad \text{[A13]}$$